1. Definitions:

$\mathbb{A} = \{a \mid a \in \mathbb{N} \wedge a = 2n\text{-}1\}$
$\mathbb{X} = \{x \mid x \in \mathbb{N} \wedge x = 2^{n-1}\}$
$\mathbb{D} = \{d \mid d \in \mathbb{N}_{\text{even}} \wedge d \equiv 1 \bmod 3\}$
$\mathbb{P} = \{p \mid p \in \mathbb{N} \wedge p \text{ is prime number}\}$

$f : \mathbb{A} \to \mathbb{D},\ a \mapsto 3a+1$
$f^{-1} : \mathbb{D} \to \mathbb{A},\ d \mapsto (d-1)/3$
$g: \mathbb{N}_{\text{even}} \to \mathbb{N},\ n \mapsto n/2$
$g^{-1} : \mathbb{N} \to \mathbb{N}_{\text{even}},\ n \mapsto 2n$

1.1 Important:

1.1.1 $\forall d \in \mathbb{D}\ \exists! a \in \mathbb{A}: d = f(a)$
1.1.2 $\forall n \in \mathbb{N}\ \exists! n \in \mathbb{N}_{\text{even}}: n = g(n)$
1.1.3 $\forall n \in \mathbb{N}: n = p_1^{r_1} \cdot p_2^{r_2} \cdot \ldots \cdot p_k^{r_k} \mid p_j \in \mathbb{P},\ r_j \in \mathbb{N}_0$ short $n = \prod p_j^{r_j}$

2. Structure:

$\mathbb{N}$ is divided into $|\mathbb{A}|$ disjoint sets

$$\mathbb{X}_{a_i} = \{a_i x \mid a_i \in \mathbb{A},\ x \in \mathbb{X}\}$$

i.e. exactly one subset $\mathbb{X}_{a_i}$ for each $a_i \in \mathbb{A}$.

Their elements are sorted according to their magnitude and thus form the sequences

$$(a_i \cdot 1,\ a_i \cdot 2,\ a_i \cdot 4,\ a_i \cdot 8,\ \ldots) \text{ short } (a_i \cdot 2^{n-1})_{n \in \mathbb{N}}$$

Each of these sequences is arranged in a line from left to right.
The root of the structure is the sequence to $\mathbb{X}_1$ (1, 2, 4, 8, …). Whenever $n \in \mathbb{D}$ applies to a member n in a row, the row with the start value $a_i = f^{-1}(n)$ begins right shifted directly below this position, also in the form $(a_i \cdot 2^{n-1})_{n \in \mathbb{N}}$. The structure can also be done in columns and rows recursively, starting at 1 recursively. The structure remains complete, i.e. all $n \in \mathbb{N}$ are contained in the structure.

3. The path of the Collatz function from each number to 1:

The iterative Collatz function "moves" to the left in each line with $g(n)$, as long as its value is even. At the start of the line a_i, it jumps upwards with $f(n)$ to the "parent" line.

The often lamented chaotic and unpredictable behaviour of the Collatz function is completely compensated for by the structure of this arrangement and is a thing of the past. The Collatz function strives predictably step by step towards 1 for any starting number.

Assertion:

Every natural number is part of this structure. Therefore, every starting value leads to 1.
There is no number $n \in \mathbb{X}_{a_i} : a_i > 1$, that leads to a cycle or grows to infinity.
We stop when we reach 1. The trivial cycle $4 \to 2 \to 1 \to 4 \to \ldots$ should not be considered further here.

Reasoning:

● **Completeness**

Each natural number is contained exactly once in this structure.
The set $\mathbb{N}$ of natural numbers can also be written as:

$$\mathbb{N} = \{n \mid n = ax, a \in \mathbb{A}, x \in \mathbb{X}\}$$

This follows directly from 1.1.3 and the associativity of multiplication in $\mathbb{N}$.

Each number $n \in \mathbb{N}$ only belongs to exactly one sequence $(a_i \cdot 2^{n-1})_{n \in \mathbb{N}}$, because the sets $\mathbb{X}a_i = \{a_i \cdot 2^{n-1} \mid a_i \in \mathbb{A}, n \in \mathbb{N}\}$ are disjoint.

Since f(a) is bijective (1.1.1), $f^{-1}(d)$ is also bijective: For every $d \in \mathbb{D}$ there is exactly one $a_i \in \mathbb{A}$ and because of

$$2 \cdot a_i < 3 \cdot a_i + 1 < 4 \cdot a$$

there is no self-mapping in $\mathbb{X}_{a_i}$ for all $a_i > 1$.

● **No loops:**

Since movements are only possible to the left and only upwards at the beginning of the line and a selfmapping in $\mathbb{X}a_i$ is not possible for $a_i>1$, no loop can occur except the 4→2→1→4→... in $\mathbb{X}_1$.
Loops over several lines in the form $a_i \to a_{\dots} \to \dots \to a_{\dots} \to a_i$ cannot occur either, since the bijectivity of f(a) ensures that each $d \in \mathbb{D}$ as well as each $a_i \in \mathbb{A}$ only occurs exactly once in the structure.

● **Exclusion of divergence by structural induction:**

Any number n leads to 1. All numbers following in the structure [$g^{-1}(n)$ and if $n \in \mathbb{D}$ $f^{-1}(n)$] also lead to 1. It follows that all elements of the structure lead to 1.

An "upward escape" is therefore ruled out due to the structure.

Conclusion:

This complete and directed arrangement means that every number $n \in \mathbb{N}$is covered and leads to 1 after a countable number of steps.

The structure presented here provides a complete and consistent model that explains and supports all known properties of the Collatz function.

Structural induction
An attempt to prove the Collatz function using classical complete induction (of $n \rightarrow n+1$) immediately encounters difficulties: The behaviour of the Collatz function for $n+1$ does not systematically depend on the behaviour for n. For example, n can lead to 1 after just a few steps, while $n+1$ initially jumps to large values before the sequence drops off again.

Instead, a structural induction that follows the arrangement of the numbers in the structure presented is a good option:

- We introduce a logical property C(n) for each number n. This property states: the number n is 1 or leads to 1 after iterative application of the Collatz function.
- If C(n) is known, then this also applies to all numbers following via g^{-1} or f^{-1} because these functions inherit the property C(n) to the numbers following in the structure.
- Since the structure includes all $n \in \mathbb{N}$, C(n) is known for all n.

Completeness of the structure
The function f(n) as well as its inverse function $f^{-1}(n)$ is bijective and $2 \cdot a < 3 \cdot a + 1 < 4 \cdot a$ applies for all $a > 1$.

It is therefore impossible that an odd number a_i or one of the following numbers above $g^{-1}(n)$ is not part of this structure.

For every odd number a, there is an even number d = f(a), which can also be represented in the form $2^n \cdot a'$. Recursively, we always get from one a to the next "superordinate" a'. This recursion only "ends" in the loop of the first line of the structure with a = 1.

Exclusion of divergence - a thought experiment with the robo-mouse "Lucie"
To visualise the impossibility of divergence due to the structure, we replace each element of the structure with a sphere. These spheres are connected by straight truncated cones, with the large opening belonging to the current value and the small opening to the next value of a Collatz sequence.

- Each sphere has at least two openings: one large and one small.
- Spheres that correspond to target values of the function f(a) also have a second small opening.

After we have replaced all the numbers with spheres, we place Lucie somewhere in the structure. Lucie is lazy: instead of squeezing into narrow openings, it always crawls into the largest opening it can find. As every large opening only leads in the direction of a smaller opening, Lucie automatically follows the course of the Collatz function.

Important: In this construction, there is no way back to a larger opening, except in the known cycle 4→2→1→4→....
If you also define sphere "1" as a sink (without a large opening), then Lucie ends up there for good.

Lucie only travels through the structure along the large openings and thus moves deterministically from one sphere to the next. This process is independent of specific numerical values - the structure alone determines the path.

Here is the recursive creation of the structure
Macro for the recursive creation of the structure
Here is a larger section of the structure